Countdown Round

2021 New Year's Invitational

Problem 1

How many 0's are at the end of $26!$?

Problem 2

What is the sum of the three smallest positive integers not divisible by 2, 3, 5, or 7?

Problem 3

6 is the sum of its proper divisors ($6 = 1 + 2 + 3$). What is the next smallest positive integer that shares this property?

(A proper divisor is a positive divisor of a number excluding itself.)

Problem 4

If the sum of an integer n and its reciprocal is 250.004, what is n?

Problem 5

Bill picks out 4 balls without replacement from a bin containing 5 red balls and 5 blue balls. Let a/b be, in simplest terms, the probability he has more blue balls than red balls. What is $a+b$?

Problem 6

Bob takes four seconds to do a **MATHCOUNTS** problem.
Bill takes five seconds to do a **MATHCOUNTS** problem.
How many more **MATHCOUNTS** problems can Bob
solve than Bill in a minute?

Problem 7

A string of digits (0-9) is *interesting* if no two consecutive digits are the same. For example, 010, 210, and 139 are *interesting* but 100 is not. How many *interesting* strings have four digits?

Problem 8

What is the sum of the digits of 1001^4?

Problem 9

The product of the positive integer divisors of n is 100. What is n?

Problem 10

What is the minimum value of

$|x - 1| + |x + 1| + |x + 3| + |x - 3|$

when x is a positive real number?

Problem 11

Bob initially has 100 cards. He sells 20% of them, and then throws away 15 cards. Finally, he sells 20% of the remaining cards. At the end, how many cards does Bob have?

Problem 12

The centroid of the triangle with vertices (4, 5), (10, 13), and (7, 15) can be expressed as the ordered pair (a, b) where a and b are both integers. What is $a+b$?

Problem 13

What is the remainder when 1024 is divided by 11?

Problem 14

Billy draws the smallest circle that contains a regular heptagon with side length 4, and then draws the biggest circle that is contained in the heptagon. The area inside the bigger circle but outside the smaller circle can be expressed as $n\pi$. Find n.

Problem 15

What is the largest possible nonzero area of a rectangle whose area is equal to double the perimeter?

Problem 16

What is the smallest integer $k > 1$ such that $n^k - n^2$ is divisible by 420 for all integers n?

Problem 17

What is the square root of $2(111^2) - 10001$?

Problem 18

What is the area of the biggest rectangle that can fit in a triangle with side lengths 13, 14, and 15?

Problem 19

What is the hundreds digit of 101^8?

Problem 20

A number is *easy* if it can be expressed as the positive difference of two consecutive positive perfect cubes. What is the sum of the first 10 *easy* numbers?

Problem 21

Billy randomly shuffles a standard card deck with 52 cards. The probability that the 10th card is a 10 can be expressed as $1/b$ where b is a positive integer. What is b?

Problem 22

Points A, B, C, D lie on a circle. If AB and CD meet at P, PA = 20, PB = 101, and PC = 5, what is PD?

Problem 23

Bill sells chicken nuggets in boxes of 8 or 5. For how many positive integers n is it impossible for Nick to buy n chicken nuggets?

Problem 24

What is the inradius of a right triangle with legs 9 and 40?

Problem 25

A positive integer is *trash* if one of its prime divisors leaves a remainder of 3 upon division by 4. How many *trash* numbers are one less than a perfect square and less than 200?